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A fixed point theorem for random operator in dislocated Quasi metric spaces

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ABSTRACT : In this paper we have proved a fixed point theorem for continues contraction mappings in dislocated Quasi Metric Spaces for random operator.

Keywords : Dislocated Quasi Metric Spaces, Spaces, Fixed Point, Random Operator

I. INTRODUCTION AND PRELIMINARIES

Probalistic functional analysis has emerged as one of the important mathematical disciplines in view of its role in analyzing probabilistic models in the applied sciences. The study of fixed points of random operators forms a central topic in this area. The Prague school of probabilistic initiated its study in the 1950. However, the research in this area flourished after the publication of the survey article of Bharucha-Reid [5]. Since then many intrusting random fixed point results and several applications have appeared in the literature; for example the work of Beg and Shahazad [2, 3], Lin [13], O'Regan [14], Papageorgiou [15] Xu [20].

In recent years, the study of random fixed points has attracted much attention. In particular random iteration schemes leading to random point of random operators have been discussed in [6, 7, 8, 10].

Banach (1922) proved Fixed Point Theorem for contraction mappings in complete metric space. It is well known as a Banach Fixed point Theorem. Dass and Gupta [11] generalized Banach's

Contaction Principle in Metric space. Also Rhoads (1077) introduced a partial ordering for various definitions contractive mappings. This objective of the note is to prove some fixed point theorem for continues contraction mapping defined by Dass and Gupta [11] and Rhoades [18] in Dislocated Quasi metric spaces. In the present paper we establish a fixed point theorem for random operator in Dislocated Quasi Metric Spaces.

Definition 1 : Let X be a nonempty set and let d: X × X → [0, ∝] be a function satisfying following conditions.

(*i*) $d(x, y) = d(y, x) = 0 \Rightarrow y = x$

(*ii*) $d(x, y) \le d(x, x) + d(z, y) \quad \forall \; x, y, z \in X$

Then d is called Dislocated Quasi metric Space on X. If d satisfies $d(x, y) = d(y, x)$ then it is called dislocated metric space.

Definition 2 : A sequence $\{X_n\}$ in Dislocated Quasi Metric Spaces (X, d) is called Cauchy Sequence if for given $\varepsilon > 0$ there exists $n_0 \in N$ such that

$\forall \; m, n > n_0 \Rightarrow \mathrm{d}(x_m, x_n) < \varepsilon$ or $d(x_n, x_m) < \varepsilon$

i.e, $\min \{d(x_m, x_n), d(x_n, x_m)\} < \varepsilon$

Definition 3 : A sequence $\{X_n\}$ Dislocated Quasi Convergence to x if

$\lim_{n \to \infty} d(x_n, x) = \lim_{n \to \infty} d(x, x_n) = 0$

In this case x is called a *dq* limit of $\{X_n\}$ we write $X_n \to X$

Definition 4 : A Dislocated Quasi Metric Space (X,d) is called complete if every Cauchy sequence in it is a *dq* convergent.

Definition 5 : Let (X, d) and (Y, d) be *dq* Metric Spaces and Let $f : X \to Y$ be a function then f is continues to $x_0 \in X$, if for each sequence $\{X_n\}$ which is $d_1 - q$ convergent to x_0 in X, the sequence $\{f(x_n)\}$ is d_2-q convergent $f(x_0)$ in Y.

Definition 6 : Let (X,d) be a *dq* metric space. A map $T : X \to$ X is called contaction if there exists $- \le x \le 1$ such than

$$d(T_x, T_y) < \lambda\, d(x, y) \;\square x, y \in X$$

Throughout this paper, (Ω, Σ) denotes a measurable space, H A Dislocated Quasi Metric Space, and C is non empty subset of H.

Measurable function 7 : A function $f : \Omega \to C$ is said to be measurable if $f^{-1}(B \cap C) \in \Sigma$ for every Borel subset B of H.

Random operator 8 : A function $f : \Omega \times C \to$ C is said to be random operator, if $F(., X) : \Omega \to C$ is measurable for every $X \in C$.

Continuous Random operator 9 : A random operator $F : \Omega \times C \to c$ is continuous

Random fixed point (*a*) : A measurable function $g : \Omega \to C$ is said to be random fixed point of the random operator $F : \Omega \times C \to$, if $F(t, g(t)) = g(t), \; \forall \; t \in \Omega$.

II. MAIN RESULT

Theorem 1 : Let (X, d) be a dq metric space and let $T : X \to$ X be continuous mapping satisfying the following condition.

$$d(T_x, T_y) \le \alpha \frac{d(y,Ty)[1+d(x,Tx)]}{1+d(x,y)} + \beta \frac{d(y,Ty)[1+d(y,Tx)]}{1+d(x,y)} + \gamma d(x,y) + \delta \frac{d(y,Ty)+d(y,Tx)}{1+d(y,Ty)d(y,Tx)}$$

$(\forall x, y \in X, \alpha > 0, \beta > 0, \gamma > 0, \delta > 0, \psi > 0, \alpha + \beta + \gamma + \delta < 1)$.

Then T has a unique fixed point.

Proof : Let $\{g_n\}$ be a sequence of function in X defined as follows

$T(\xi, g_n(\xi)) = g_{n+1}(\xi)$, Consider,

$d(g_n(\xi), g_{n+1}(\xi)) = d(T(\xi, g_{n-1}(\xi), T(\xi, g_n(\xi))$

$$\le \alpha \frac{d(g_n(\xi), T(\xi, g_n(\xi))[1+d(g_{n-1}(\xi), T(\xi, g_{n-1}(\xi)]}{1+d(g_n(\xi), g_{n-1}(\xi))} +$$

$$\beta \frac{d(g_n(\xi), T(\xi, g_n(\xi))[1+d(g_n(\xi), T(\xi, g_{n-1}(\xi))]}{d(g_n(\xi), g_{n-1}(\xi))}$$

$+ \gamma\, d(g_n(\xi), g_{n-1}(\xi))$

$$+ \delta \frac{d(g_n(\xi), T(\xi, g_n(\xi)) + d(g_n(\xi), T(\xi, g_{n-1}(\xi))}{1+d(g_n(\xi), T(\xi, g_n(\xi))[d(g_n(\xi), T(\xi, g_{n-1}(\xi))]}$$

$d(g_n(\xi), g_{n+1}(\xi)) \le$

$$\alpha \frac{d(g_n(\xi), g_{n+1}(\xi))[1+d(g_{n-1}(\xi), x_n g_n(\xi))]}{1+d(g_n(\xi), g_{n-1}(\xi))}$$

$$+ \beta \frac{d(g_n(\xi), g_{n+1}(\xi))[1+d(g_n(\xi))]}{1+d(g_n(\xi), g_{n-1}(\xi))}$$

$+ \gamma\, d(g_{n-1}(\xi), g_n(\xi)) +$

$$\delta \frac{d(g_n(\xi), T(\xi, g_n(\xi)) + d(g_n(\xi), T(\xi, g_{n-1}(\xi))}{1+d(g_n(\xi), T(\xi, g_n(\xi))[d(g_n(\xi), T(\xi, g_{n-1}(\xi)]}$$

$$= \alpha \frac{d(g_n(\xi), g_n(\xi))[1+d(g_{n-1}(\xi), g_n(\xi))]}{1+d(x,y)}$$

$$+ \beta \frac{d(g_n(\xi), g_{n+1}(\xi))}{1+d(g_{n-1}(\xi), g_n(\xi))} + \gamma\, d(g_{n-1}(\xi), g_n(\xi))$$

$$+ \delta \frac{d(g_n(\xi), g_{n+1}(\xi))[d(g_n(\xi), g_n(\xi))]}{1+d(g_n(\xi), g_{n+1}(\xi))[d(g_n(\xi), g_n(\xi))]}$$

$\Rightarrow (1 - \alpha - \beta - \delta) d(g_n(\xi), g_{n+1}(\xi)) \le \gamma\, d(g_{n-1}(\xi), g_n(\xi))$

$$\Rightarrow d(g_n(\xi), g_{n+1}(\xi)) \le \frac{\gamma}{(1-\alpha-\beta-\delta)} d(g_{n-1}(\xi), g_n(\xi)),$$

where $k = \dfrac{\gamma}{(1-\alpha-\beta-\delta)}$ with $0 \le k \le 1$

Then $d(g_n(\xi), g_{n+1}(\xi)) \le k\, d(g_{n-1}(\xi), g_n(\xi))$

On further decomposing we can write

$d(g_{n-1}(\xi), g_n(\xi)) \le k\, d(g_{n-2})(\xi), g_{n-1}(\xi))$ and finally we can write $d(g_n(\xi), g_{n+1}(\xi)) \le k^2\, d(g_{n-2})(\xi), g_{n-1}(\xi))$

On continuing this process n times

$d(g(\xi), g_{n+1}(\xi)) \le k^2\, d(g_0(\xi), g_1(\xi))$

Since $0 \le k \le 1$ and $n \to \infty$, $k^n \to 0$

Hence $\{g_n(\xi)\}$ is a dislocated Quasi sequence in the complete dislocated Quasi metric space X.

Thus $\{g_n(\xi)\}$ dislocated Quasi sequence converges to some $\{P(\xi)\}$

Since T is Continous we have

$T(\xi, P(\xi)) = \lim_{n\to 0} T(\xi, g_n(\xi)) = \lim_{n\to 0} g_{n+1}(\xi) = P(\xi)$

Thus $T(\xi, P(\xi)) = P(\xi)$

Thus T has fixed point.

The uniqueness is trivial.

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